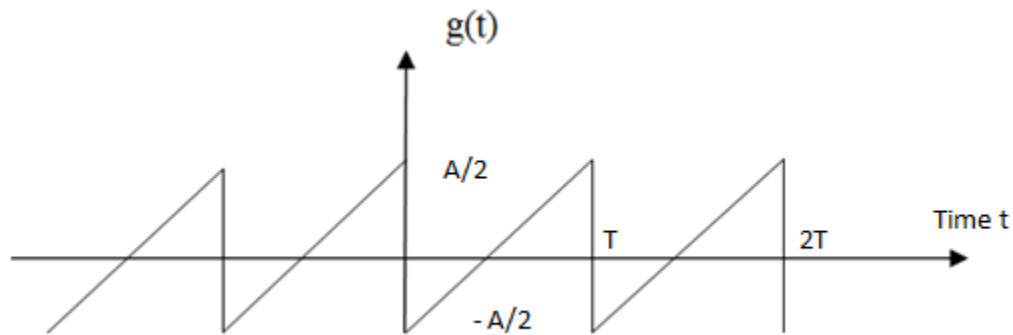# Problem Set 4

Pulse Code Modulation

1. A baseband signal $m(t)$ has a spectral range that extends from 0 to 25 kHz. The signal is sampled at a rate of f_s samples/sec. Find the Nyquist rate for this signal.

2. Find the Nyquist rate for the multi-tone signal $m(t) = 2\cos(200\pi t) + 2\cos(300\pi t) + 2\cos(400\pi t)$.

3. The Fourier transform of a signal $m(t)$ is given as:

$$M(f) = \begin{cases} A & -100 \leq f \leq 100 \\ 0 & |f| > 100 \end{cases}$$

The signal is sampled at a rate of 150 Samples/sec.

   a. Sketch the spectrum of the sampled signal

   b. If the sampled signal is admitted to an ideal low-pass filter with a bandwidth of 100 Hz. Sketch the spectrum of the filter output.

   c. Is there a distortion? Explain the reason

4. Find the Nyquist rate for the signal $m(t) = A sinc(1000t)$.

5. The signal $x(t) = \cos(2\pi t)$ is uniformly sampled at a rate of 20 samples/sec. The samples are applied to an 8-level uniform quantizer with a dynamic range (-1, 1) V and a step size of 0.25 V.

   a. Plot the sampled signal over one cycle of the message
   b. Plot the quantizer output over one cycle of the message
   c. Repeat Part b if the signal applied to the quantizer is $g(t) = 0.25 \cos(2\pi t)$
   d. Comment on the results of Parts b and c

6. The signal $m(t) = \cos(1000\pi t)$ is to transmitted using a PCM system (a system composed of a sampler, quantizer, and binary encoder).

   a. If sampling is done at the Nyquist rate and a uniform quantizer with 32 levels is employed, what is the resulting data rate in bits/sec and the resulting SQNR

   b. Find the SQNR if the signal is sampled at 1.5 times the Nyquist rate

7. Draw the output of the DM given that the input corresponds to x(t)=1.1t + 0.05 when the input is sampled at t=0, 1, 2, 3, 4, 5, … and $\Delta = 1$.

8. Repeat Problem 7, if $\Delta = 0.5$

9. Repeat Problem 7, if $\Delta = 1.5$. Comment on the results of Problems 7, 8, and 9.
10. Design a 16-level uniform quantizer for an input signal with a dynamic range of $(-10, +10)V$.
    a. Find the quantizer output and the quantization error for an input sample of 1.2 V.
    b. Find the binary representation corresponding to the sample -2.63 assuming natural binary encoding is used.
    c. Find the average SQNR
11. Plot the SQNR, in dB, derived in class versus L (number of quantization levels) for L=2, 4, 8, 16, 32, 64. What are your conclusions?
12. Reconstruct a staircase signal at the receiver side of a delta demodulator with $\Delta = 0.1V$, when the received data sequence is 1 1 1 1 0 0 1 1 1 1 0 1 0 1 1 1.
13. Reconstruct a staircase signal at the receiver side of a delta demodulator with $\Delta = 0.1V$, when the received data sequence is 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0. Comment on the nature of the message signal
14. The triangular signal g(t), shown in the figure below, is applied to an 8-level uniform quantizer with a range of (-A/2, A/2). Find the SQNR. The noise error can still be assumed to be a uniform random variable over $(-\frac{\Delta}{2}, \frac{\Delta}{2})$.

15. Design a 7-level uniform quantizer for the input signal with a dynamic range of $\pm 10V$. Note that the quantized values include the zero level (the quantized values are $0, \pm\frac{\Delta}{2}, \pm 3\frac{\Delta}{2}, \pm 5\frac{\Delta}{2}$).